The Furuta Pendulum Experiment

1. Objectives

The Furuta pendulum experiment is very similar to an experiment where a thin rod has to be balanced on a cart. This is probably the most popular experiment in control theory and is ideally suited to teach the methods of a second course in control system design. The rotary version has the advantage of being smaller and easier to handle.

In a first course on control, the students normally learn to design PID-type controllers which are of great practical importance for plants which have well damped poles with the possible exception of a pole at the origin. Typical examples are level control, pressure control, motor control, temperature control or control of a concentration. The rotary pendulum does not fit into this list and requires more advanced methods of controller design. These methods were invented in the late sixties under the name “state space methods”.

In the simplest case, the full state is fed back and the controller is designed by pole placement. The students learn to place the poles suitably and to understand the role of the time constant of the differentiators. A slightly more advanced design technique yields the linear quadratic regulator. If only one deflection angle is measured, a Luenberger observer has to be constructed. Another type of observer studied in this experiment is the famous Kalman filter. A LQ controller in combination with a Kalman filter yields a LQG controller (or $\mathcal{H}_2$ controller. The characteristic property of all these methods is that appropriate weights have to be chosen. The students learn how to do this and can see how the choice influences the performance of the controlled rotary pendulum. They also learn to apply classical concepts like the Nyquist curve.

The main task of the experiment is to be a platform to teach advanced methods of linear control. Despite of this, the experiment is also well suited for the implementation of nonlinear controllers. The task of such a controller is to swing-up the pendulum. One controller of this kind is implemented in the prepared files. For the students or the instructor it should be no serious problem to implement other nonlinear controllers described in the literature.

2. Plant and Simulation Model

The control variable is the input voltage coming from the computer and the output variables are the deflection angle $\psi$ of the horizontal rod and the deflection angle $\phi$ of the vertical rod. If friction is neglected, the transfer functions for the hanging pendulum has a pole pair on the imaginary axis:

$$G_{\psi u}(s) = \frac{-k_1}{s^2 + \omega_i^2}$$
For the inverted pendulum, one obtains
\[ G_{yu}(s) = \frac{k_1}{(s + \omega_2)(s - \omega_1)} \]
\[ G_{wu}(s) = k_2 \frac{(s + \omega_2)(s - \omega_2)}{s^2(s + \omega_1)(s - \omega_1)} \]

These transfer functions are used for controller design. The full simulation model contains the nonlinearity, viscous and Coulomb friction, limitations and so on. Figure 3 shows the SIMULINK model of the closed-loop system. It will be used for non real-time simulation as well as for real-time simulation. The controller block contains two identical controllers: One serves for the simulation and the other is connected to hardware.

Some of the experiments dealing with the plant are:

- Step responses for small input signals to illustrate the dynamics of the transfer functions
- Step responses for large input signals to compare the linerized model with the nonlinear model
- Step responses to show the effect of Coulomb friction on the dynamics
- Responses to sinusoidal inputs to demonstrate the practical application of Bode plots.

3. Controllers

Various controllers have to be designed and tested. The typical experiments for the closed-loop systems are:

- Generation of a step response for a (constant) command
- Tracking a non constant reference variable
- Rejection of a disturbance.

**PD control of the horizontal rod.** The first controller that has to be designed has the task to position the horizontal rod while the vertical rod is neglected. This can be done with the tools of a first course on control system design. Depending on the design, the controller shifts the weakly damped pole pair of the hanging pendulum, but it remains weakly damped. Thus the wish arises to design a better controller which suppresses additionally the oscillations of the hanging pendulum.

**Pole placement controller for the hanging pendulum.** A controller which puts the horizontal bar in the desired position without causing oscillations of the hanging pendulum can be designed by pole
placement. This controller feeds back the complete state. The rates are obtained by a differentiator.

**Design of an estimator.** If only one angle is measured, the other has to be estimated by an observer. Two observers have to be designed and tested:

- A Luenberger observer
- A Kalman filter.

**Design of a controller with only one measurement.** Such a controller can be obtained by putting together a pole placement controller and an observer. The influence of the design parameters on the closed-loop system will be found out experimentally.

**Design of a pole placement controller for the inverted pendulum.** Two pole placement controllers are designed, namely:

- Stabilization only
- Stabilization and tracking.

**Design of $H_\infty$ controllers.** Such a controller will be designed for the hanging pendulum as well as for the inverted pendulum. The design procedure consists of two steps:

- Design of a LQ controller
- Design of a Kalman filter.

The students learn to chose suitable weights and have the possibility to try out their role experimentally.

It can be observed that for the inverted pendulum a limit cycle occurs. The reason for this limit cycle is the Coulomb friction in the bearings. This can be shown by switching on and off the Coulomb friction in the simulation model.

**Design of a nonlinear controller.** A controller for swinging-up the pendulum will be designed. If the pendulum is near the horizontal position, the nonlinear controller will be switched out and a linear controller will stabilize the pendulum.

### 4. Analysis Tools

In order to get a deeper understanding of controller design and the fundamental properties of the plants and the closed-loop systems, the PendCon SW includes various m.files in which the required analysis is prepared. For the plants, the following plots can be generated:

- Pole-zero maps of the plant (in dependency of the linearization point)
- Bode plots of the plant.

For controller synthesis and analysis, it is possible to obtain the following plots:

- A plot of the Nyquist curve
- Bode plot of the loop transfer function
- Bode plots of the other transfer functions of the closed-loop system. By this way, it is possible to determine the bandwidth of the closed-loop system, to study disturbance rejection properties of the CLS or to analyze the control effort. Also, robustness properties can be studied with these plots.
- Pole-zero maps of the CLS.

By this way, the students have the possibility to learn how the design parameters for the various controllers effect the poles of the CLS and the Bode plots of the transfer function of the CLS (this is necessary for $H_\infty$ synthesis).

### 5. Simulation Results

The following figures show some simulation results which are obtained from the experiments. In Fig. 4 the red (blue) lines represent the measured (simulated) quantities, respectively. Figure 4 shows the tracking of the command for the deflection angle $\phi$ of the link mounted on the motor shaft. The angle that the other pendulum makes with the horizontal line is $\psi$ and this pendulum has to be hold in the upward position.

Another experiment is concerned with the swing-up for the Furuta pendulum. The swing-up controller works in such a way that which each swing the $\psi$-pendulum will be brought closer and closer to the perpendicular position. If this pendulum is close enough to the perpendicular position, the swing-up controller will be switched off and a linear controller takes on control. Figure 5 shows the measured quantities for a swing-up experiment.

**Video Clips.** We finally mention the video clips belonging to the experiments which can be found on www.pendcon.de. These clips show:

- Step response, tracking a command and disturbance rejection for the hanging pendulum
- Tracking and disturbance rejection for the inverted pendulum
- Swinging-up the pendulum.
Fig. 4: Tracking a command

Fig. 5: Swinging-up the pendulum